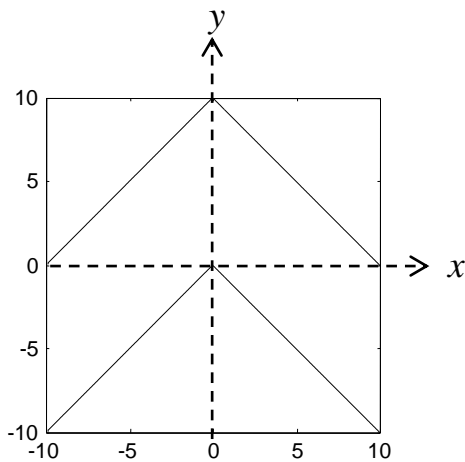


**QAT Sample Questions (SET 3)**

1. Let  $f(x) = \frac{(x+2)(x+3)}{(x+1)(x+6)}$ . What is  $f'(0)$ ?
- A.  $5/6$
  - B.  $-4/3$
  - C.  $1$
  - D.  $-1/3$
  - E.  $2/3$

2. The following graph (solid lines only) shows level curves of which function? (A level curve of  $f(x, y)$  is a set of points  $(x, y)$  such that  $f(x, y)$  is constant.)



- A.  $f(x, y) = |y| - x$
  - B.  $f(x, y) = |x| + y$
  - C.  $f(x, y) = |x|y$
  - D.  $f(x, y) = 10|x + y|$
  - E.  $f(x, y) = 10|x - y|$
3. Suppose  $f(x, y) = \sqrt{2x + y}$ . Which of the following is the equation of the plane that is tangent to the graph of  $f$  (plotted on the  $x, y, z$  axes) at the point  $(4, 1)$ ?
- A.  $z = \frac{1}{6}x + \frac{1}{3}y + 2$
  - B.  $z = \frac{1}{6}x + \frac{1}{3}y + 1.5$
  - C.  $z = \frac{1}{3}x + \frac{1}{6}y - 1.5$
  - D.  $z = \frac{1}{3}x + \frac{19}{6}y - 1.5$
  - E.  $z = \frac{1}{3}x + \frac{1}{6}y + 1.5$

4. If  $\frac{dy}{dx} = y \sec^2 x$  and  $y = 5$  when  $x = 0$ , then  $y =$
- A.  $e^{\tan x} + 4$
  - B.  $e^{\tan x} + 5$
  - C.  $5e^{\tan x}$
  - D.  $\tan x + 5$
  - E.  $\tan x + 5e^x$
5. A candle is burning so that when its height is  $h$  cm, it is burning at a rate of  $3\sqrt{h}$  cm per hour. If the initial height of the candle is 36 cm, how long will it take for the candle to burn down completely?
- A. 4 hours
  - B. 5 hours
  - C. 6 hours
  - D. 7 hours
  - E. 8 hours
6. Let  $A = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$ . Let  $A^T$  denote the transpose of  $A$ . What is the determinant of  $(A^T)^2$ ?
- A.  $-2$
  - B.  $-9$
  - C.  $0$
  - D.  $2$
  - E.  $3$
7. Let  $f(x) = 1 + x$  and  $g(x) = x^2$ . Which of the following polynomials is not a linear combination of  $f(x)$  and  $g(x)$ ?
- A.  $1 + 2x + 3x^2$
  - B.  $5 + 5x + \frac{1}{2}x^2$
  - C.  $3 + 3x - x^2$
  - D.  $-2 - 2x + 4x^2$
  - E.  $1 + x - x^2$

8. Two balls are chosen (without replacement) at random from an urn consisting of three red balls, two blue balls, and one pink ball. What is the probability that at least one blue ball is chosen?
- A.  $1/5$
  - B.  $2/5$
  - C.  $3/5$
  - D.  $2/15$
  - E.  $8/15$
9. An urn consists of three white balls and two red balls. Frank and Jane alternately randomly choose a ball (with replacement). The first person who gets the same color as the latest ball chosen by the other person wins the game. Assume that Frank starts first, and he cannot win from the first ball. What is the probability that Jane wins this game?
- A.  $13/19$
  - B.  $13/25$
  - C.  $19/25$
  - D.  $13/15$
  - E.  $1/2$

Problem 10-11

Assume that the daily returns of an asset is normally distributed with mean  $\mu$  and variance  $\sigma^2$ . Let  $X_1, X_2, \dots, X_{10}$  be ten independent sample of the asset returns. Define the sample average  $\bar{X}$  and the sample variance  $S^2$  as follows:

$$\bar{X} = \frac{1}{10} \sum_{i=1}^{10} X_i, \quad S^2 = \frac{1}{9} \sum_{i=1}^{10} (X_i - \bar{X})^2$$

Answer the following questions.

10. What is the distribution of  $\bar{X}$ ?
- A. Normal with mean  $\mu/10$  and variance  $\sigma^2/10$
  - B. Normal with mean  $\mu/10$  and variance  $\sigma^2$
  - C. Normal with mean  $\mu$  and variance  $\sigma^2/10$
  - D. Normal with mean  $\mu$  and variance  $\sigma^2/100$
  - E. Normal with mean  $\mu/10$  and variance  $\sigma^2/100$
11. The sample variance  $S^2$ , after scaling by an appropriate constant, has which of the following distribution?
- A. Normal
  - B. Chi-square, degree 9
  - C. Chi-square, degree 10
  - D. Student-t, degree 9
  - E. Student-t, degree 10

12. Let  $X$  and  $Y$  be two random variables with variances 1 and 2, respectively. If the covariance of  $X$  and  $Y$  is  $-0.75$ , which of the following has the lowest variance?
- A.  $X$
  - B.  $-X$
  - C.  $0.5X + 0.5Y$
  - D.  $0.5X - 0.5Y$
  - E.  $0.6X + 0.4Y$
13. Many firms encourage employees to participate in assistance programs that enable employees, free of charge, to improve their physical fitness and reduce stress. These programs increase worker productivity, reduce absenteeism, and lessen insurance costs for employee health care. Therefore, these programs benefit both the company and the employee.

Which of the following, if true, most significantly strengthens the conclusion above?

- A. Physical fitness reduces employees' risk of heart disease.
  - B. Physical fitness is the most popular program.
  - C. Employee assistance programs require company to hire more employees to monitor the programs offered.
  - D. Studies have shown that stress management course is not effective.
  - E. Physical injuries can result from entering a physical fitness program without warming up the body.
14. Leafleting and speechmaking on government property should be outlawed. Radicals and fanatics have no rights to use public property when peddling their unsavory views.

The argument above would be more effective if which of the following assumptions holds?

- A. radicals and fanatics prefer using public property when disseminating their views.
- B. legal restrictions that apply to one group need not apply equally to all.
- C. the general restrictions that apply to one group need not apply equally to all.
- D. political activity that interferes with the orderly functioning of government should not be protected by law.
- E. all those who leaflet and make speeches on government property are radicals and fanatics.

Use the following facts to answer questions 15 - 16

The office staff of the Mango Corporation presently consists of three executive vice presidents or EVP (A, B, and C) and five vice presidents or VP (D, E, F, G, and H). Management is planning to open a new office in another city using three VP and two EVP of the present staff. The following guidelines were established to set up the new office:

1. D and G have not been on speaking terms for many months. They should not go together.
2. Since D and F have been competing for promotion, they should not go together.
3. EVP A and C are constantly finding fault with one another and should not go together.
4. C and E function well alone but not as a team. They should be separated.

15. Who has to go to the new office?

- A. B
- B. D
- C. E
- D. G
- E. H

16. If C is sent to the new office, which member of the staff cannot go with C?

- A. B
- B. D
- C. F
- D. G
- E. H

17. Sales of mobile phones have increased drastically over the last year. In order to take advantage of this increase, Banana Company plans to expand production of its current model of mobile phone, while continuing its already very extensive advertising of this product.

Which of the following, if true, provides most support for the view that Banana cannot increase its sales of mobile phone by executing the plan above?

- A. Banana's inventory of mobile phones has declined over the last year.
- B. Although it sells all the mobile phones that it produces, Banana's share of all mobile phone sales has declined over the last year.
- C. Banana's mobile phone is the top brand of mobile phone sold last year.
- D. Sales of Banana's mobile phones have fallen in the last year.
- E. Advertising has made the brand name of Banana's mobile phones well known, but not many customers know that Banana Company owns this brand.

List of symbols for Questions 18-21

Symbol	Description
$\sim$	Negation
$\wedge$	Logical 'and'
$\vee$	Logical 'or'
$P \Rightarrow Q$	$P$ is sufficient for $Q$ . $Q$ is necessary for $P$
$P \Leftrightarrow Q$	$P$ is equivalent to $Q$
$\forall x$	For all $x$
$\exists x$	There exists an $x$

18. Which of the following pairs of propositions are not equivalent?

- A.  $\sim(P \vee Q) \Leftrightarrow R$ ,  $(\sim P \Leftrightarrow R) \wedge (\sim Q \Leftrightarrow R)$
- B.  $\sim(P \wedge Q)$ ,  $\sim P \vee \sim Q$
- C.  $\sim(P \Rightarrow (Q \wedge R))$ ,  $\sim(P \Rightarrow Q) \vee \sim(P \Rightarrow R)$
- D.  $P \Rightarrow (Q \Rightarrow R)$ ,  $\sim(P \wedge Q) \vee R$
- E. All pairs are equivalent

19. Which of the following statements has the conclusion that is logical?
- A. Tigers are carnivorous. Elephants are herbivorous. Therefore, elephants are bigger than tigers.
  - B. All birds can fly, and penguins are birds. Therefore, penguins can fly.
  - C. If Pat wins a first prize lottery, Pat can afford a new car. Since Pat did not win the first prize lottery, Pat can't afford a new car.
  - D. Both B. and C.
  - E. There is no correct answer

The following sentences are for questions 20 - 21:

- a:  $n$  is divisible by 5
- b:  $n$  is divisible by 10
- c:  $n$  is divisible by 20
- d:  $n = 40$
- e:  $n$  is even and divisible by 5
- f:  $n^2$  is divisible by 5

20. Which conditions above are necessary for  $n$  to be divisible by 10?
- A. a, b, and c
  - B. b, c, and d
  - C. b, e, and f
  - D. a, c, and e
  - E. a, b, e, and f
21. Which conditions above are sufficient for  $n$  to be divisible by 10?
- A. a, b, c, d
  - B. b, c, d, e
  - C. c, d, e, f
  - D. a, b, e, f
  - E. b, c, d, e, f

**Solutions:**

1. D

2. B

3. E

4. C

5. A

6. C

7. A

8. C

9. A

10. C

11. B

12. E

13. A

14. E

15. A

16. B

17. D

18. A

19. B

20. E

21. B